Math 201–Midterm (Summer 15) B. Shayya and H. Yamani

- Please write your **section number** on your booklet.
- Please answer each problem on the **indicated page(s)** of the booklet. Any part of your answer not written on the indicated page(s) will not be graded.
- Unjustified answers will receive little or no credit.

Problem 1 (answer on page **1** of the booklet.)
(8 pts each) Which of the following sequences converge, and which diverge? Find the limit of each convergent sequence.

$$a_n = \frac{n^{2000}}{2^n} \qquad b_n = \frac{\sin(e^{10n}-1)}{100n+(n!)} \qquad c_n = \sqrt[n]{n!}$$

Problem 2 (answer on pages **2** and **3** of the booklet.)
(9 pts each) Which of the following series converge, and which diverge? When possible, find the sum of the series.

(i) $\sum_{n=1}^{\infty}\left(\frac{3^n}{5^{n-2}}+\frac{(-1)^n}{2^{n+1}}\right)$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n^4+n-1}}{n^8+n^5+3}$ (iii) $\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{\sqrt{n}}\right)$

Problem 3 (answer on page **4** of the booklet.)
(15 pts) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{\ln n}{n}(x-5)^n.$$

Problem 4 (answer on page **5** of the booklet.)
(i) (7 pts) Use ASET to estimate $\ln(1.01)$ with an error of magnitude less than 10^{-6}. Does your estimate tend to be an overestimate or an underestimate?
(ii) (7 pts) Use Taylor's theorem to estimate the error resulting from the approximation $e^x \approx 1+x$ for $|x| \le 0.01$.
(iii) (7 pts) Use Taylor's theorem to estimate the error resulting from the approximation $\sqrt{1+x} \approx 1+\frac{x}{2}$ for $|x| \le 0.01$.

Problem 5 (answer on page **6** of the booklet.)
(a) (7 pts) Does

$$\lim_{(x,y)\to(0,0)}\frac{x^3y}{x^2+y^4}$$

exist? Why or why not?
(b) (7 pts) Does

$$\lim_{(x,y)\to(0,0)}\frac{xy}{2x+y}$$

exist? Why or why not?
(c) (7 pts) Suppose $f(x,y,z)$ is a differentiable function of three variables with

$$f_x(x,y,z)=3x^2+y^2+2xz, \qquad f_y(x,y,z)=2xy, \quad \text{and} \quad f_z(x,y,z)=x^2.$$

Let

$$x = \frac{r}{s}, \qquad y = r + \ln s, \qquad z = r + s^2, \quad \text{and} \quad w = f(x, y, z).$$

Find $\partial w/\partial r$ and $\partial w/\partial s$ at the point $(r, s) = (2, 1)$.

Problem 6 (answer on page 7 of the booklet.)

(12 pts) Match each of the following equations with the surface it defines.

1. $x = y^2 - z^2$
2. $x = z^2 - y^2$
3. $z^2 + x^2 - y^2 = 1$
4. $z^2 + 4y^2 - 4x^2 = 4$
5. $x^2 + 4z^2 = y^2$
6. $y^2 + z^2 = x^2$
7. $9y^2 + z^2 = 16$
8. $x^2 + 2z^2 = 8$
9. $9x^2 + 4y^2 + 2z^2 = 36$
10. $x^2 + y^2 + 4z^2 = 10$
11. $x = -y^2 - z^2$
12. $z = -4x^2 - y^2$